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# A Theory for the Small Unsymmetric Deformations of Cylindrical Shells

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A THEORY FOR THE SMALL UNSYMMETRIC DEFORMATIONS  
OF CYLINDRICAL SHELLS

Edward L. Reiss

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# A Theory For The Small Unsymmetric Deformations of Cylindrical Shells

## 1. Introduction

In reference 1 approximate theories to determine the rotationally symmetric deformations of "thick" cylindrical shells were developed in a systematic manner from the exact three dimensional theory of linear elasticity for an isotropic and homogeneous material (hereafter referred to as the exact theory). These approximate theories were obtained by a method that is in a sense, a generalization of the boundary layer expansion technique used by Friedrichs<sup>2</sup> and later Friedrichs and Dressler<sup>3</sup> in a study of the bonding of plates.\* However the scaling and expansion techniques differ somewhat from those in reference and 3.

In this paper, the method presented in reference 1 is extended to include unsymmetric deformations of cylindrical shells. Similar scaling and expansion procedures are employed and the present results specialize to those of reference 1 for rotationally symmetric deformations. The basic element in the approximation is a "thin" shell theory. The "thick" shell theories are then obtained as "corrections" to the thin shell theory. We find that the so called Donnell theory of cylindrical shells<sup>5</sup>, which appears naturally as a consequence of our expansion procedures, is the appropriate thin shell theory. A summary of the thick shell theory is given in Section 6.

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\*In this connection see also reference 4.



## 2. Formulation.

In terms of the cylindrical coordinate system,  $(r, \theta, x)$ , a cylindrical shell of uniform thickness  $2h$ , radius  $R$  and length  $L$  is defined as an elastic body bounded by the coaxial surfaces  $r = R \pm h$  and the planes  $x = 0$  and  $x = L$ . The inner and outer surfaces of the cylinder,  $r = R - h$  and  $r = R + h$ , respectively, are subjected only to normal pressures. The edges of the cylinder formed by the annular regions  $x = 0, L, R - h \leq r \leq R + h$ , are submitted to normal and shear forces. The applied forces are arbitrary provided they form a system in equilibrium.

Our purpose is to obtain, in a systematic manner from the exact theory, two dimensional or "shell" theories which approximate the deformations of cylindrical bodies with "small" values of  $h/R$ . It is convenient to introduce the dimensionless variables:

$$(1) \quad \xi = \frac{x}{R\epsilon^a} = \frac{x}{\sqrt{Rh}}, \quad \phi = \frac{\theta}{\epsilon^a} = \frac{\theta}{(h/R)^{1/2}}, \quad \zeta = \frac{r-R}{R\epsilon^{2a}} = \frac{r-R}{h},$$

where,  $a$ , is a positive integer and  $\epsilon$  is the dimensionless parameter,

$$(2) \quad \epsilon = \left(\frac{h}{R}\right)^{1/2a}.$$

The inner and outer surfaces are thus given by  $\zeta = \pm 1$ , respectively, while the ends of the cylinder correspond to  $\xi = 0, L'$  where,  $L' = \frac{L}{R\epsilon^a} = \frac{L}{\sqrt{Rh}}$ . A more general scaling procedure than (1) and (2) may be used. However, this leads to "unreasonable" shell theories which are discussed in reference 1

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5. The fifth step is to implement the solution.

6. The sixth step is to evaluate the solution.

7. The seventh step is to monitor the solution.

8. The eighth step is to report the results.

9. The ninth step is to reflect on the process.

10. The tenth step is to conclude.

11. The eleventh step is to communicate the results.

12. The twelfth step is to document the process.

13. The thirteenth step is to review the process.

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for the special case of rotationally symmetric deformations.

Attention in this paper is therefore restricted to the scaling in (1) and (2). The integer,  $a$ , provides some flexibility in the choice of the parameter,  $\epsilon$ . For simplicity of presentation we set  $a = 1$  and hence,\*

$$\epsilon = (h/R)^{1/2}.$$

Dimensionless stress and displacement components are defined by dividing the physical stresses by Young's modulus and the physical displacements by  $R$ . Using these dimensionless quantities and the variables (1) and (2), the equations of the exact theory neglecting body forces are, with an obvious notation: equilibrium equations,

$$\begin{aligned} \sigma_{z,\zeta} + \epsilon \sigma_{zx,\xi} + g(\zeta)[\sigma_z - \sigma_\theta] + \epsilon^{-1} \sigma_{z\theta,\phi} &= 0, \\ (3) \quad \sigma_{zx,\zeta} + \epsilon \sigma_{x,\xi} + g(\zeta)[\sigma_{zx} + \epsilon^{-1} \sigma_{x\theta,\phi}] &= 0, \\ \sigma_{z\theta,\zeta} + \epsilon \sigma_{x\theta,\xi} + g(\zeta)[2\sigma_{z\theta} + \epsilon^{-1} \sigma_{\theta,\phi}] &= 0; \end{aligned}$$

stress displacement relations (Hooke's Law),

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\*In principle, no difficulty is encountered if,  $a$ , is an arbitrary integer. However the calculations become more cumbersome; see reference 1.

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$$\begin{aligned}
(4) \quad & u, \xi = \varepsilon[\sigma_x - \nu(\sigma_\theta + \sigma_z)], \\
& v, \phi + \varepsilon w = \varepsilon(1 + \varepsilon^2 \zeta)[\sigma_\theta - \nu(\sigma_x + \sigma_z)], \\
& u, \phi + (1 + \varepsilon^2 \zeta)v, \xi = \varepsilon(1 + \varepsilon^2 \zeta)2(1 + \nu)\sigma_{x\theta}, \\
& w, \zeta = \varepsilon^2[\sigma_z - \nu(\sigma_x + \sigma_\theta)], \\
& u, \zeta + \varepsilon w, \xi = \varepsilon^2 2(1 + \nu)\sigma_{zx}, \\
& v, \zeta - g(\zeta)[v - \varepsilon^{-1}w, \phi] = \varepsilon^2 2(1 + \nu)\sigma_{z\theta};
\end{aligned}$$

Compatibility Equations,

$$\begin{aligned}
& \Delta \sigma_x + \varepsilon^2 \Omega, \xi \xi = 0, \\
& \Delta \sigma_z + \Omega, \zeta \zeta - 2g^2(\zeta)(\sigma_z - \sigma_\theta + 2\varepsilon^{-1}\sigma_{z\theta, \phi}) = 0, \\
& \Delta \sigma_\theta + g(\zeta) \Omega, \zeta + 2g^2(\zeta)(\sigma_z - \sigma_\theta + 2\varepsilon^{-1}\sigma_{z\theta, \phi} \\
& \quad + \frac{\varepsilon^{-2}}{2} \Omega, \phi \phi) = 0, \\
(5) \quad & \Delta \sigma_{zx} + \varepsilon \Omega, \xi \zeta - g^2(\zeta)(\sigma_{zx} + 2\varepsilon^{-1}\sigma_{x\theta, \phi}) = 0, \\
& \Delta \sigma_{z\theta} + \varepsilon^{-1}g(\zeta) \Omega, \zeta \phi \\
& \quad + 2g^2(\zeta)[-2\sigma_{z\theta} + \varepsilon^{-1} \frac{\partial}{\partial \phi}(\sigma_z - \sigma_\theta - \frac{1}{2} \Omega)] = 0, \\
& \Delta \sigma_{x\theta} + g(\zeta) \Omega, \xi \phi + g^2(\zeta)(-\sigma_{x\theta} + \varepsilon^{-1}\sigma_{zx, \phi}) = 0,
\end{aligned}$$

where  $\nu$  is Poisson's ratio and,

$$\begin{aligned}
g(\zeta) &= \frac{\varepsilon^2}{1 + \varepsilon^2 \zeta}, & \Omega &= \frac{1}{1 + \nu}[\sigma_x + \sigma_\theta + \sigma_z], \\
\Delta &\equiv \frac{\partial^2}{\partial \zeta^2} + \varepsilon^2 \frac{\partial^2}{\partial \xi^2} + g(\zeta) \frac{\partial}{\partial \zeta} + \varepsilon^{-2} g^2(\zeta) \frac{\partial^2}{\partial \phi^2}.
\end{aligned}$$



In (3.5) we have employed the conventional notation that an independent variable appearing as a subscript following a comma denotes differentiation with respect to that variable. Thus,

$$\sigma_{z\Omega, \zeta} = \frac{\partial \sigma_{z\Omega}}{\partial \zeta} .$$

A complete formulation of the elastic problem for the cylinder (Formulation A) consists of (3) and (4) and appropriate boundary conditions on the surfaces and ends of the cylinder. A second and equivalent formulation (Formulation B) consists of (3) and (5) with appropriate boundary conditions. These conditions may be obtained by specifying the applied forces in the following manner:

$$(6a) \quad \begin{cases} \sigma_{zx}(\xi, \phi, \pm 1; \epsilon) = \sigma_{z\Omega}(\xi, \phi, \pm 1; \epsilon) = 0 , \\ \sigma_z(\xi, \phi, 1; \epsilon) = p_o(\xi, \phi; \epsilon), \quad \sigma_z(\xi, \phi, -1; \epsilon) = p_I(\xi, \phi; \epsilon); \end{cases}$$

$$(6b)^* \quad \begin{cases} \sigma_x(0, \phi, \zeta; \epsilon) = \bar{\sigma}(\phi, \zeta; \epsilon), \quad \sigma_{zx}(0, \phi, \zeta; \epsilon) = \bar{\tau}(\phi, \zeta; \epsilon) , \\ \sigma_{x\Omega}(0, \phi, \zeta; \epsilon) = \bar{\tau}(\phi, \zeta; \epsilon) ; \end{cases}$$

and conditions similar to (6b) on  $\xi = L$ . Here  $p_o(\xi, \phi; \epsilon)$  and  $p_I(\xi, \phi; \epsilon)$  are, respectively, the applied normal forces on the outer and inner surfaces of the cylinder.  $\bar{\sigma}(\phi, \zeta; \epsilon)$ ,  $\bar{\tau}(\phi, \zeta; \epsilon)$  and

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\*  $\bar{\tau}(\phi, \pm 1; \epsilon) = \bar{\tau}(\phi, \pm 1; \epsilon) = 0$  for continuity.

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$\bar{F}(\phi, \zeta; \epsilon)$  are the forces applied to the ends of the cylinder. We further impose the unessential but convenient restriction that,

$$(7) \quad p_0 \neq p_I.$$

Formulation A is employed in the following sections to discuss the deformations in the "interior" of the cylinder. To examine the deformations in the "boundary layer"<sup>\*</sup> we use Formulation B in sections 4 and 5.

### 3. The Interior Problems.

We assume that each component of stress and displacement, indicated by the generic symbol,  $\sigma(\xi, \phi, \zeta; \epsilon)$ , can be represented, asymptotically, as a power series in  $\epsilon$ :

$$(8) \quad \sigma(\xi, \phi, \zeta; \epsilon) \sim \sum_{n=0}^{\infty} \sigma^n(\xi, \phi, \zeta) \epsilon^n$$

where  $\sigma^n(\xi, \phi, \zeta) = 0$  if  $n < 0$ . The functions  $\sigma^n(\xi, \phi, \zeta)$  are called the interior stress coefficients or the interior displacement coefficients of order  $n$  whichever the case may be. We further assume that the prescribed forces in (6) can be expanded in power series in  $\epsilon$ :

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<sup>\*</sup>The term "boundary layer" refers to a narrow volume of the cylinder adjacent to and including the ends where stresses and deflections may change rapidly in the  $\xi$  direction. The remaining part of the cylinder is called the "interior".

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$$(9.a) \quad \left\{ \begin{array}{c} p_0(\xi, \phi; \varepsilon) \\ p_I(\xi, \phi; \varepsilon) \end{array} \right\} = \sum_{n=0}^{\infty} \left\{ \begin{array}{c} p_0^n(\xi, \phi) \\ p_I^n(\xi, \phi) \end{array} \right\} \varepsilon^n ;$$

$$(9.b) \quad \left\{ \begin{array}{c} \bar{\sigma}(\phi, \zeta; \varepsilon) \\ \bar{\tau}(\phi, \zeta; \varepsilon) \\ \bar{t}(\phi, \zeta; \varepsilon) \end{array} \right\} = \sum_{n=0}^{\infty} \left\{ \begin{array}{c} \bar{\sigma}^n(\phi, \zeta; \varepsilon) \\ \bar{\tau}^n(\phi, \zeta; \varepsilon) \\ \bar{t}^n(\phi, \zeta; \varepsilon) \end{array} \right\} \varepsilon^n .$$

Expansions of the form (8) are substituted into the equilibrium equations (3) and the stress displacement relations (4). Coefficients of the same powers of  $\varepsilon$  are equated yielding a system of differential equations satisfied by the interior stress and displacement coefficients of all orders. The coefficients of  $\varepsilon^n$  are:

$$(10a) \quad \sigma_{z, \zeta}^n + \sigma_{zx, \xi}^{n-1} + \sum' (\sigma_z^i - \sigma_\Omega^i) + \sum'' \sigma_{z\Omega}^i = 0 ,$$

$$(10b) \quad \sigma_{zx, \zeta}^n + \sigma_{x, \xi}^{n-1} + \sum' \sigma_{zx}^i + \sum'' \sigma_{x\Omega}^i = 0 ,$$

$$(10c) \quad \sigma_{z\Omega, \zeta}^n + \sigma_{x\Omega, \xi}^{n-1} + 2 \sum' \sigma_{z\Omega}^i + \sum'' \sigma_\Omega^i = 0 ;$$

$$(11a) \quad u_{, \zeta}^n = \sigma_x^{n-1} - \nu(\sigma_\Omega^{n-1} + \sigma_z^{n-1}) ,$$

$$(11b) \quad v_{, \phi}^n + w^{n-1} = \sigma_\Omega^{n-1} - \nu(\sigma_x^{n-1} + \sigma_z^{n-1})$$

$$+ \zeta[\sigma_\Omega^{n-3} - \nu(\sigma_x^{n-3} + \sigma_z^{n-3})] ,$$

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that proper record-keeping is essential for transparency and accountability, particularly in financial matters. The text suggests that organizations should implement robust systems to track every aspect of their operations, from procurement to sales, to ensure that all data is reliable and accessible.

2. The second part of the document addresses the challenges of data management in a rapidly changing environment. It highlights the need for flexible and scalable solutions that can adapt to new technologies and evolving business requirements. The author argues that investing in modern data infrastructure is not just a technical necessity but a strategic imperative for long-term success.

3. The third part of the document explores the role of data in decision-making. It argues that data-driven insights are crucial for identifying trends, opportunities, and risks. By leveraging analytics and machine learning, organizations can make more informed decisions and optimize their performance. The text also touches upon the importance of data security and privacy, noting that these are key concerns for stakeholders and regulators alike.

4. The fourth part of the document discusses the importance of collaboration and communication in data management. It suggests that cross-functional teams are essential for breaking down silos and ensuring that data is shared and used effectively across the organization. The author emphasizes that clear communication and collaboration are key to maximizing the value of data and achieving organizational goals.

5. The fifth part of the document provides a summary of the key points discussed and offers some final thoughts on the future of data management. It concludes that while the challenges are significant, the opportunities are vast, and organizations that embrace data-driven approaches will be better positioned to thrive in the future.

$$(11c) \quad v_{,\xi}^n + \zeta v_{,\xi}^{n-2} + u_{,\phi}^n = 2(1 + \nu)[\sigma_{x\Omega}^{n-1} + \zeta \sigma_{x\Omega}^{n-3}] \quad ,$$

$$(11d) \quad w_{,\zeta}^n = \sigma_z^{n-2} - \nu(\sigma_x^{n-2} + \sigma_\Omega^{n-2}) \quad ,$$

$$(11e) \quad u_{,\zeta}^n + w_{,\xi}^{n-1} = 2(1 + \nu)\sigma_{zx}^{n-2} \quad ,$$

$$(11f) \quad v_{,\zeta}^n + \zeta v_{,\zeta}^{n-2} - v^{n-2} + w_{,\phi}^{n-1} = 2(1 + \nu)[\sigma_{z\Omega}^{n-2} + \zeta \sigma_{z\Omega}^{n-4}] \quad .$$

Here,

$$\begin{aligned} \sum' A^i &= \sum_{i+2} \sum_{(j+1)=n} (-1)^j \zeta^j A^i \\ &\qquad\qquad\qquad i, j \geq 0 \quad . \\ \sum'' A^i &= \sum_{i+2} \sum_{j+1=n} (-1)^j \zeta^j A_{,\phi}^i \end{aligned}$$

Appropriate boundary conditions on  $\zeta = \pm 1$  for the interior stress and displacement coefficients are obtained by substituting (9a) and expansions of the form (8) into (6a) and equating coefficients of like powers of  $\epsilon$ . This yields, for the coefficients of  $\epsilon^n$ ,

$$(12a) \quad \sigma_{zx}^n(\xi, \phi, \pm 1) = \sigma_{z\Omega}^n(\xi, \phi, \pm 1) = 0 \quad ,$$

$$(12b) \quad \left\{ \sigma_z^n(\xi, \phi, 1), \sigma_z^n(\xi, \phi, -1) \right\} = \left\{ p_o^n(\xi, \phi), p_I^n(\xi, \phi) \right\}$$

Equations to determine each interior coefficient are obtained from (10-12). We first solve (11a,b,c) for  $\sigma_x^n$ ,  $\sigma_\Omega^n$  and  $\sigma_{x\Omega}^n$ ,

1. The first part of the report is a general introduction to the subject.

2. The second part is a detailed description of the methods used.

3. The third part is a discussion of the results obtained.

4. The fourth part is a conclusion and a list of references.

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$$\begin{aligned}
\sigma_x^n &= \frac{1}{1-\nu^2} [u_{,\xi}^{n+1} + \nu(v_{,\phi}^{n+1} + w^n)] \\
&\quad + \frac{\nu}{1-\nu} \sigma_z^n - \frac{\nu}{1-\nu^2} [\sigma_\Omega^{n-2} - \nu(\sigma_x^{n-2} + \sigma_z^{n-2})] \zeta, \\
(13) \quad \sigma_\Omega^n &= \frac{1}{1-\nu^2} [v_{,\phi}^{n+1} + w^n + \nu u_{,\xi}^{n+1}] \\
&\quad + \frac{\nu}{1-\nu} \sigma_z^n - \frac{1}{1-\nu^2} [\sigma_\Omega^{n-2} - \nu(\sigma_x^{n-2} + \sigma_z^{n-2})] \zeta, \\
\sigma_{x\Omega}^n &= \frac{1}{2(1+\nu)} [v_{,\xi}^{n+1} + u_{,\phi}^{n+1} + \zeta v_{,\xi}^{n-1}] - \zeta \sigma_{x\Omega}^{n-2}.
\end{aligned}$$

If  $n = 0$ , it follows from (11), recalling that  $\sigma^n = 0$  if  $n < 0$ , that,

$$u_{,\xi}^0 = u_{,\zeta}^0 = v_{,\phi}^0 = v_{,\zeta}^0 = v_{,\xi}^0 + u_{,\phi}^0 = w_{,\zeta}^0 = 0.$$

Therefore,

$$(14) \quad u^0(\xi, \phi, \zeta) = v^0(\xi, \phi, \zeta) = 0, w^0(\xi, \phi, \zeta) = w^0(\xi, \phi),$$

where  $w^0(\xi, \phi)$  is an arbitrary function to be subsequently determined.\* Similarly we find from (10b,c) and (12a) that,

$$(15a) \quad \sigma_{zx}^0(\xi, \phi, \zeta) = \sigma_{z\Omega}^0(\xi, \phi, \zeta) = 0.$$

Using this result in conjunction with (12b) and (7), Eq. (10a) yields,

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\*Here we have assumed that  $u^0(\xi, \phi, \zeta)$  is periodic in  $\phi$  and have neglected rigid body deformations.

Figure 1. Schematic representation of the experimental design. The subjects were divided into two groups: the control group (C) and the experimental group (E). The control group (C) was divided into two subgroups: the control group (C) and the control group (C). The experimental group (E) was divided into two subgroups: the experimental group (E) and the experimental group (E).

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$$(15b) \quad \sigma_z^n = p_o^n = p_I^n = 0, \quad \text{if } n < 2.$$

Returning to (11d) we see that,

$$(16) \quad w^n(\xi, \phi, \zeta) = w^n(W, \phi) , \quad \text{if } n < 2.$$

Employing this last result, (14) and (15a), (11e,f) yield,

$$u_{,\zeta}^n + w_{,\xi}^{n-1} = v_{,\zeta}^n + w_{,\phi}^{n-1} = 0, \quad \text{if } n = 1, 2,$$

which implies that,

$$(17) \quad \left. \begin{aligned} u^n(\xi, \phi, \zeta) &= -w_{,\xi}^{n-1} \zeta + U^n(\xi, \phi) , \\ v^n(\xi, \phi, \zeta) &= -w_{,\phi}^{n-1} \zeta + V^n(\xi, \phi) , \end{aligned} \right\} \quad \text{if } n = 1, 2 .$$

Here  $U^n(\xi, \phi)$  and  $V^n(\xi, \phi)$  are arbitrary functions which are determined later.

Substitution of (15) and (17) into (13) gives the first two stress-displacement relations for  $\sigma_x^n$ ,  $\sigma_\phi^n$  and  $\sigma_{x\phi}^n$  in terms of  $U^1, V^1$  and  $W^0$ , and  $U^2, V^2$  and  $W^1$ :

$$(18) \quad \left. \begin{aligned} \sigma_x^n(\xi, \phi, \zeta) &= s_x^n(\xi, \phi) + \bar{s}_x^n(\xi, \phi) \zeta , \\ \sigma_\phi^n(\xi, \phi, \zeta) &= s_\phi^n(\xi, \phi) + \bar{s}_\phi^n(\xi, \phi) \zeta , \\ \sigma_{x\phi}^n(\xi, \phi, \zeta) &= s_{x\phi}^n(\xi, \phi) + \bar{s}_{x\phi}^n(\xi, \phi) \zeta , \end{aligned} \right\} \quad \text{if } n = 0, 1 ,$$





where, \*\*

$$(19a) \quad \begin{cases} S_x^n(\xi, \phi) = \frac{1}{1-\nu^2} [U_{,\xi}^{n+1} + \nu(V_{,\phi}^{n+1} + W^n)] , \\ S_\Omega^n(\xi, \phi) = \frac{1}{1-\nu^2} [V_{,\phi}^{n+1} + W^n + \nu U_{,\xi}^{n+1}] , \\ S_{x\Omega}^n(\xi, \phi) = \frac{1}{2(1+\nu)} [V_{,\xi}^{n+1} + U_{,\phi}^{n+1}] , \end{cases}$$

$$(19b) \quad \begin{cases} \bar{S}_x^n(\xi, \phi) = \frac{-1}{1-\nu^2} [W_{,\xi\xi}^n + \nu W_{,\phi\phi}^n] , \\ \bar{S}_\Omega^n(\xi, \phi) = - \frac{1}{1-\nu^2} [W_{,\phi\phi}^n + \nu W_{,\xi\xi}^n] , \\ \bar{S}_{x\Omega}^n(\xi, \phi) = - \frac{1}{1+\nu} W_{,\phi\xi}^n . \end{cases}$$

We now obtain two systems of differential equations to determine  $U^1, V^1$  and  $W^0$ , and  $U^2, V^2$  and  $W^1$ . Substituting (15a) and (18) into (10b,c) with  $n = 1$  or  $2$ , integrating with respect to  $\xi$  and using the surface conditions (12a) there results,

$$(20) \quad S_{x,\xi}^n + S_{x\Omega,\phi}^n = 0, \quad S_{x\Omega,\xi}^n + S_{\Omega,\phi}^n = 0, \quad \text{if } n = 0, 1,$$

and

$$(21) \quad \left. \begin{aligned} \sigma_{zx}^n(\xi, \phi, \zeta) &= \frac{1}{2} [\bar{S}_{x,\xi}^{n-1} + \bar{S}_{x\Omega,\phi}^{n-1}] (1 - \zeta^2) , \\ \sigma_{z\Omega}^n(\xi, \phi, \zeta) &= \frac{1}{2} [\bar{S}_{\Omega,\phi}^{n-1} + \bar{S}_{x\Omega,\xi}^{n-1}] (1 - \zeta^2) , \end{aligned} \right\} \text{if } n = 1, 2 .$$

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\*\*The coefficients defined in (19) are proportional to the conventional stress resultants and couples.<sup>1,6,7</sup>

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that proper record-keeping is essential for transparency and accountability, particularly in financial matters. The text outlines various methods for collecting and organizing data, including the use of spreadsheets and specialized software. It also mentions the need for regular audits to ensure the integrity of the information.

2. The second part of the document focuses on the legal and regulatory aspects of record-keeping. It references relevant laws and regulations that govern the collection, storage, and disposal of records. The text highlights the importance of understanding these requirements to avoid legal penalties and ensure compliance. It also discusses the role of record-keeping in legal proceedings and the potential consequences of non-compliance.

3. The third part of the document addresses the practical challenges of implementing a record-keeping system. It discusses the need for clear policies and procedures, as well as the importance of training staff on the correct use of the system. The text also mentions the importance of ensuring the security of the records and the need for regular backups to prevent data loss. It concludes by emphasizing the long-term benefits of a well-implemented record-keeping system, such as improved efficiency and better decision-making.

Equations (20) supply four of the required differential equations. Equations (21) are the stress displacement relations for the first two non-vanishing interior transverse shear stress coefficients. The remaining two differential equations and stress displacement relations for the first two non-vanishing  $\sigma_z^n$  are obtained by substituting (15), (18) and (21) into (10a) with  $n = 2$  or  $3$ . Integrating the resulting equation once with respect to  $\zeta$  and using the surface conditions (12b) we find that,

$$(22) \quad \bar{s}_{x,\xi\xi}^n + 2\bar{s}_{x\phi,\phi\xi}^n + \bar{s}_{\phi,\phi\phi}^n - 3s_{\phi}^n = 3(p_I^{n+2} - p_O^{n+2}),$$

$$\text{if } n = 0, 1 \quad ,$$

and

$$(23) \quad \sigma_z^n(\xi, \phi, \zeta) = \frac{3}{2}(-s_{\phi}^{n-2} + p_O^n - p_I^n)(\zeta - \frac{\zeta^3}{3}) - \frac{1}{2}s_{\phi}^{n-2}(1 - \zeta^2) \\ + s_{\phi}^{n-2}\zeta + (p_O^n + p_I^n) \quad , \quad \text{if } n = 2, 3 \quad .$$

It is now assumed, with no loss of generality, that both  $p_O^2(\xi, \phi)$  and  $p_I^2(\xi, \phi)$  do not simultaneously identically vanish. This implies that the normal forces applied to the inner and outer surfaces are of order of magnitude  $\epsilon^2 = \frac{h}{R}$ .

To summarize, we have obtained by a systematic expansion procedure relations expressing each of the first two non-vanishing interior stress coefficients in terms of the first two non-vanishing interior displacement coefficients. In addition appropriate differential equations to determine these displacement coefficients have been obtained. These results may be written in



a more familiar form. Substituting (19) into (20) and (22) and setting  $n = 0$  gives the differential equations of the "zeroth order interior problem" as:

$$(24) \quad \begin{aligned} U^1_{,\xi\xi} + \frac{1-\nu}{2} U^1_{,\phi\phi} + \frac{1+\nu}{2} V^1_{,\phi\xi} + \nu W^0_{,\xi} &= 0, \\ V^1_{,\phi\phi} + \frac{1-\nu}{2} V^1_{,\xi\xi} + \frac{1+\nu}{2} U^1_{,\phi\xi} + W^0_{,\phi} &= 0, \end{aligned}$$

$$\Delta^2 W^0 + 3(V^1_{,\phi} + W^0 + \nu U^1_{,\xi}) = 3(1 - \nu^2)(p_0^2 - p_I^2) ;$$

while from (18), (19), (21) and (23) the stress displacement relations of the zeroth order interior problem are,

$$(25) \quad \begin{cases} \sigma^0_x(\xi, \phi, \zeta) = s^0_x(\xi, \phi) + \bar{s}^0_x(\xi, \phi)\zeta, \\ \sigma^0_\phi(\xi, \phi, \zeta) = s^0_\phi(\xi, \phi) + \bar{s}^0_\phi(\xi, \phi)\zeta, \\ \sigma^0_{x\phi}(\xi, \phi, \zeta) = s^0_{x\phi}(\xi, \phi) + \bar{s}^0_{x\phi}(\xi, \phi)\zeta, \end{cases}$$

$$(26) \quad \begin{cases} \sigma^1_{zx}(\xi, \phi, \zeta) = -\frac{1}{2(1-\nu^2)}(W^0_{,\xi\xi\xi} + W^0_{,\phi\phi\xi})(1 - \zeta^2), \\ \sigma^1_{z\phi}(\xi, \phi, \zeta) = -\frac{1}{2(1-\nu^2)}(W^0_{,\xi\xi\phi} + W^0_{,\phi\phi\phi})(1 - \zeta^2), \\ \sigma^2_z(\xi, \phi, \zeta) = \frac{1}{2(1-\nu^2)}[\Delta^2 W^0(\zeta - \zeta^3/3) \\ + (W^0_{,\phi\phi} + \nu W^0_{,\xi\xi})(1 - \zeta^2) \\ + 2(V^1_{,\phi} + W^0 + \nu U^1_{,\xi})\zeta] + (p_0^2 + p_I^2). \end{cases}$$



Here  $\Delta^2$  is the two-dimensional biharmonic operator

$$\Delta^2 = \frac{\partial^4}{\partial \xi^4} + 2 \frac{\partial^4}{\partial \xi^2 \partial \phi^2} + \frac{\partial^4}{\partial \phi^4}$$

and the coefficients in (25) are given in (19). The differential equations and stress displacement relations of the "first order interior problem" are similarly obtained from (18) - (23). These expressions will be referred to as Eq. (27). They are identical with (24) - (26) provided that a, l, is added to the superscript of each term. If the deformations are rotationally symmetric then all the previous results reduce to those of reference 1.

The differential equations (24) and the stress displacement relations (25) are, in our notation, identical with those of the thin shell theory of Donnell.<sup>5</sup> Equations (26), which are not given in the Donnell theory, provide "thick shell corrections" to this thin shell theory. Thus the Donnell equations appear in a natural way as part of the first approximation of the exact theory.\*

The differential equations and stress displacement relations for interior problems of order two or greater are obtained, in a similar fashion, from (10 - 13). Because of the complexity of the analysis they are not explicitly shown here. We merely remark that,  $\sigma_x^2, \sigma_\theta^2, \sigma_{x\theta}^2, u^3$  and  $v^3$  are cubics in  $\zeta$ , while  $w^2$  is quadratic in  $\zeta$ .

To complete the formulation of the interior problems, boundary conditions appropriate to the differential equations (20) and (22) and (27) are required. These boundary conditions are systematically

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\*This does not necessarily imply that other thin shell theories,<sup>5-8</sup> could not serve as first approximations to the exact equations if different expansion procedures or parameters are employed.





derived in sections 4 and 5 from the exact theory. We observe that all the results for the interior problems are obtained without reference to the edge boundary conditions, (6b).

#### 4. Formulation of the Boundary Layer Problem.

The specific dependence of the interior stress coefficients  $\sigma_x^n$ ,  $\sigma_{zx}^n$  and  $\sigma_{x\alpha}^n$  on  $\xi$  is given by the interior analysis.\* However the boundary conditions on the ends of the shell, (6b), imply that these coefficients are arbitrary functions of  $\xi$ . Therefore, the expansions (8) cannot satisfy these conditions and if they represent the solution they do so only in a region away from the ends, i.e. (8) are not uniformly valid. To obtain expansions that are valid up to and including the end  $\xi = 0$ , we introduce a "boundary layer coordinate",  $\eta$ , by "stretching" the variable  $\xi$  so that the resulting expansions may represent the solution uniformly:†

$$(28) \quad \eta = \frac{\xi}{\epsilon} = \frac{x}{h} \quad .$$

Thus by making  $\epsilon$  sufficiently small, every neighborhood of the end in the  $\xi$  variable corresponds to an arbitrarily large one in the  $\eta$  variable. The  $\phi$  variable is not stretched. This implies that the rapid variations that occur near the boundary take place only in the  $\xi$  direction, i.e., the direction normal to the boundary.

\*For example, see (25) and (26).

†For descriptions of these boundary layer methods see References 1 - 4. In the following we confine our attention to the end  $\xi = 0$ . An identical analysis is valid for the end  $\xi = L$ .

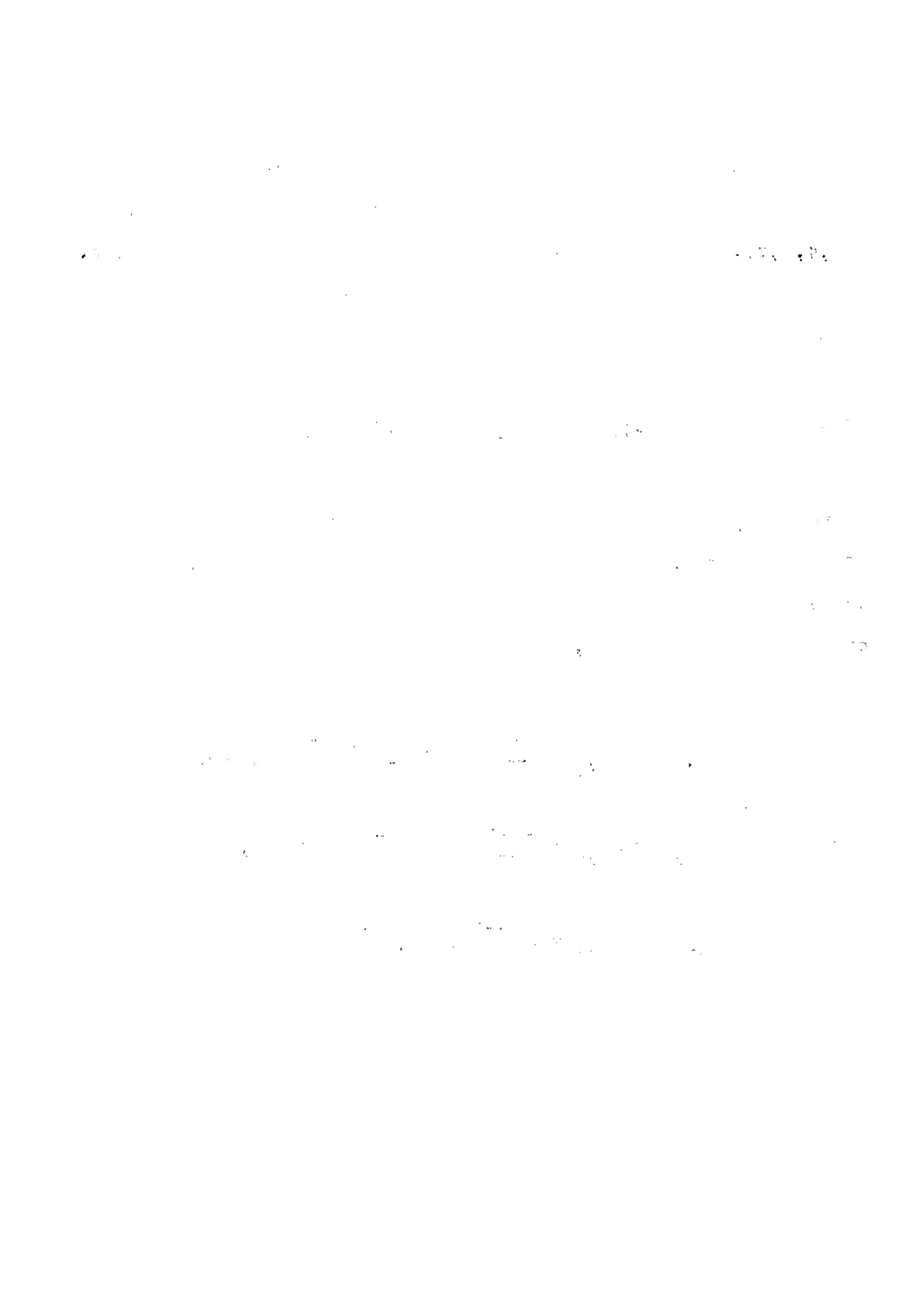


We introduce (28) into formulation B of Section 2 and define the boundary layer stresses indicated by the generic symbol,  $f(\eta, \phi, \zeta; \varepsilon)$ , as the physical stresses divided by Young's modulus. Each of the boundary layer stresses is assumed to be representable by a power series in  $\varepsilon$ :

$$(29) \quad f(\eta, \phi, \zeta; \varepsilon) \sim \sum_{i=0}^{\infty} f^i(\eta, \phi, \zeta) \varepsilon^i ,$$

where  $f^i(\eta, \phi, \zeta)$  are the boundary layer stress coefficients and  $f^i = 0$  if  $i < 0$ . Substituting (28) and (29) into the equilibrium and compatibility equations (3) and (5) and equating coefficients of the same powers of  $\varepsilon$ , we obtain for the coefficients of  $\varepsilon^n$ :

$$(30) \quad \left\{ \begin{array}{l} f_{z,\zeta}^n + f_{zx,\eta}^n + \sum' (f_z^i - f_\theta^i) + \sum'' f_{z\theta}^i = 0 \\ f_{zx,\zeta}^n + f_{x,\eta}^n + \sum' f_{zx}^i + \sum'' f_{x\theta}^i = 0 , \\ f_{z\theta,\zeta}^n + f_{x\theta,\eta}^n + 2 \sum' f_{z\theta}^i + \sum'' f_\theta^i = 0 ; \end{array} \right.$$



$$\begin{aligned}
& \nabla^2 f_x^n + \Gamma_{,\eta\eta}^n + \sum^{*'} f_x^i = 0 , \\
& \nabla^2 f_z^n + \Gamma_{,\zeta\zeta}^n + \sum^{*'} f_z^i - 2 \sum^{'''} (f_z^i - f_\Omega^i) + 4 \sum^{IV} f_{z\Omega}^i = 0 , \\
& \nabla^2 f_\Omega^n + \sum^{*'} [f_\Omega^i + \Gamma^i] + 2 \sum^{'''} (f_z^i - f_\Omega^i) + 4 \sum^{IV} f_{z\Omega}^i = 0 , \\
(31) \quad & \nabla^2 f_{zx}^n + \Gamma_{,\eta\zeta}^n + \sum^{*'} f_{zx}^i - \sum^{'''} f_{zx}^i - 2 \sum^{IV} f_{x\Omega}^i = 0 , \\
& \nabla^2 f_{x\Omega}^n + \sum^{*'} f_{x\Omega}^i + \sum^{''} \Gamma_{,\eta}^i - \sum^{'''} f_{x\Omega}^i + \sum^{IV} f_{zx}^i = 0 , \\
& \nabla^2 f_{z\Omega}^n + \sum^{*'} f_{z\Omega}^i + \sum^{''} \Gamma_{,\zeta}^i - 4 \sum^{'''} f_{z\Omega}^i \\
& \quad + \sum^{IV} [2(f_z^i - f_\Omega^i) - \Gamma^i] = 0 .
\end{aligned}$$

Here, the binomial expansions of  $g(\zeta)$  and  $g^2(\zeta)$  are employed,  $\sum^{*}$  and  $\sum^{''}$  are defined following Eq's. (11), and

$$\left. \begin{aligned}
\sum^{*'} A^i &\equiv \sum_{i+2(j+1)=n} \sum_{j=n} (-1)^j \zeta^j [A_{,\zeta}^i + (j+1)A_{,\phi\phi}^i] , \\
\sum^{'''} A^i &\equiv \sum_{i+2(j+2)=n} \sum_{j=n} (-1)^j (j+1) \zeta^j A^i , \\
\sum^{IV} A^i &\equiv \sum_{i+2j+3=n} \sum_{j=n} (-1)^j (j+1) \zeta^j A_{,\phi}^i ,
\end{aligned} \right\} i, j \geq 0 ,$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \zeta^2} ,$$

$$\Gamma^n = \frac{1}{1+\nu} [f_x^n + f_z^n + f_\Omega^n] .$$

1. The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

2. The second part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

3. The third part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

4. The fourth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

5. The fifth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

6. The sixth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

7. The seventh part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

8. The eighth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

9. The ninth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

10. The tenth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

11. The eleventh part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

12. The twelfth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

13. The thirteenth part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

To complete the formulation, the boundary layer stresses must satisfy the boundary conditions at the ends of the cylinder and on the inner and outer surfaces. In addition these stresses should approach or "match" the interior stresses for "large" values of  $\eta$ , i.e., "small" values of  $\varepsilon$ .

From (6), (9) and (29) we find that on the boundaries of the cylinder the  $f^n(\eta, \phi, \zeta)$  satisfy the following conditions:

$$(32) \quad \begin{aligned} f_{zx}^n(\eta, \phi, \pm 1) &= f_{z\theta}^n(\eta, \phi, \pm 1) = 0, \\ \{f_z^n(\eta, \phi, 1), f_z^n(\eta, \phi, -1)\} &= \sum_{j=0}^{n-1} \left\{ P_{0j}^{n-j}, P_{1j}^{n-j} \right\} \eta^j; \end{aligned}$$

$$(33) \quad \begin{aligned} f_x^n(0, \phi, \zeta) &= \bar{\rho}^n(\phi, \zeta), \quad f_{zx}^n(0, \phi, \zeta) = \bar{c}^n(\phi, \zeta), \\ f_{x\theta}^n(0, \phi, \zeta) &= \bar{t}^n(\phi, \zeta). \end{aligned}$$

Here the functions  $P_{0j}^i(\phi)$  and  $P_{1j}^i(\phi)$  are given by\*:

$$\begin{aligned} P_{0j}^i(\phi) &= \begin{cases} \frac{1}{j!} \frac{\partial p_0^i(0, \phi)}{\partial \xi^j}, & j = 0, 1, \dots, \\ 0, & j < 0 \end{cases} \\ P_{1j}^i(\phi) &= \begin{cases} \frac{1}{j!} \frac{\partial p_1^i(0, \phi)}{\partial \xi^j}, & j = 0, 1, \dots, \\ 0, & j < 0. \end{cases} \end{aligned}$$

---

\*See (9a).





They are the coefficients obtained by expanding  $p_0^i$  and  $p_I^i$  in a Taylor series in  $\xi$  about  $\xi = 0$ . The formulas, (32), follow by substituting (28) in this series and then using (29).

The "matching conditions", or the asymptotic form of the boundary layer stress coefficients, for a fixed  $\phi$ , are written as:

$$(34) \quad \lim_{\eta \rightarrow \infty} f^n(\eta, \phi, \zeta) = \sigma^{*n}(\eta, \phi, \zeta), \quad n = 0, 1, \dots,$$

where,

$$(35a) \quad \sigma^{*n}(\eta, \phi, \zeta) = \sum_{j=0}^n s_j^{n-j}(\phi, \zeta) \eta^j,$$

and

$$(35b) \quad s_j^i(\phi, \zeta) = \begin{cases} \frac{1}{j!} \frac{\partial^j \sigma^i(0, \phi, \zeta)}{\partial \xi^j}, & i \geq 0, \\ 0 & i < 0. \end{cases}$$

The matching conditions are derived by assuming that in the neighborhood of  $\xi = 0$  each interior stress coefficient can be expanded in a Taylor series in  $\xi$ . The conditions (34) and (35) follow\* by substituting (28) into this expansion and using (8) and (29).

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\*See Ref. 1 for details of the derivation.



## 5. Analysis of the Boundary Layer Problem.

In analyzing the boundary layer problem, Eq's. (30 - 35), we frequently use certain integral relations deduced from the solutions of Dirichlet problems for the harmonic and biharmonic equations on the semi-infinite strip,  $D : |\zeta| \leq 1, \infty > \eta \geq 0$ . These relations are given in the Appendix.

For every value of  $n$  equations (30 - 35) separate into two distinct systems which we denote as Problem P and Problem T. Problem P involves only  $f_x^n, f_z^n, f_\Delta^n$  and  $f_{zx}^n$ , while Problem T considers the remaining two stress coefficients. For  $n = 0$  we have for Problem P the first two of (30) and the first four of (31):

$$(36) \quad f_{z,\zeta}^0 + f_{zx,\eta}^0 = 0, \quad f_{zx,\zeta}^0 + f_{x,\eta}^0 = 0;$$

$$(37) \quad \begin{cases} \nabla^2 f_x^0 + \Gamma_{,\eta\eta}^0 = 0, & \nabla^2 f_z^0 + \Gamma_{,\zeta\zeta}^0 = 0, \\ \nabla^2 f_\Delta^0 = 0, & \nabla^2 f_{zx}^0 + \Gamma_{,\eta\zeta}^0 = 0. \end{cases}$$

The appropriate boundary and matching conditions follow from (32 - 35):

$$(38) \quad \begin{aligned} f_{zx}^0(\eta, \phi, \pm 1) &= f_z^0(\eta, \phi, \pm 1) = 0, \\ f_x^0(0, \phi, \zeta) &= \bar{\sigma}^0(\phi, \zeta), \quad f_{zx}^0(0, \phi, \zeta) = \bar{\tau}^0(\phi, \zeta); \end{aligned}$$

the first of these is the fact that the majority of the specimens are of the same sex.

The second is the fact that the majority of the specimens are of the same age.

The third is the fact that the majority of the specimens are of the same species.

The fourth is the fact that the majority of the specimens are of the same sex.

The fifth is the fact that the majority of the specimens are of the same age.

The sixth is the fact that the majority of the specimens are of the same species.

The seventh is the fact that the majority of the specimens are of the same sex.

The eighth is the fact that the majority of the specimens are of the same age.

The ninth is the fact that the majority of the specimens are of the same species.

The tenth is the fact that the majority of the specimens are of the same sex.

The eleventh is the fact that the majority of the specimens are of the same age.

The twelfth is the fact that the majority of the specimens are of the same species.

The thirteenth is the fact that the majority of the specimens are of the same sex.

The fourteenth is the fact that the majority of the specimens are of the same age.

The fifteenth is the fact that the majority of the specimens are of the same species.

The sixteenth is the fact that the majority of the specimens are of the same sex.

The seventeenth is the fact that the majority of the specimens are of the same age.

The eighteenth is the fact that the majority of the specimens are of the same species.

The nineteenth is the fact that the majority of the specimens are of the same sex.

The twentieth is the fact that the majority of the specimens are of the same age.

The twenty-first is the fact that the majority of the specimens are of the same species.

The twenty-second is the fact that the majority of the specimens are of the same sex.

The twenty-third is the fact that the majority of the specimens are of the same age.

The twenty-fourth is the fact that the majority of the specimens are of the same species.

The twenty-fifth is the fact that the majority of the specimens are of the same sex.

The twenty-sixth is the fact that the majority of the specimens are of the same age.

The twenty-seventh is the fact that the majority of the specimens are of the same species.

The twenty-eighth is the fact that the majority of the specimens are of the same sex.

$$(39) \quad \lim_{\eta \rightarrow \infty} \left\{ f_x^0, f_\theta^0, f_z^0, f_{zx}^0 \right\} = \left\{ [s_x^0(0, \phi) + \bar{s}_x^0(0, \phi)\zeta], \right. \\ \left. [s_\theta^0(0, \phi) + \bar{s}_\theta^0(0, \phi)\zeta], [0], [0] \right\}.$$

In the boundary value problem (36 - 39) the variable  $\phi$  is a parameter. This problem can be reduced to one for the biharmonic equation on D by introducing the reduced boundary layer stress coefficients,  $F^0(\eta, \phi, \zeta)$ , and an associated Airy stress function,  $\chi^0(\eta, \phi, \zeta)$ , in the following manner:

$$(40) \quad F_x^0 \equiv f_x^0 - \lim_{\eta \rightarrow \infty} f_x^0 = \chi_{,\zeta\zeta}^0, \quad F_z^0 \equiv f_z^0 = \chi_{,\eta\eta}^0, \\ F_{zx}^0 \equiv f_{zx}^0 = -\chi_{,\eta\zeta}^0, \quad F_\theta^0 \equiv f_\theta^0 - \lim_{\eta \rightarrow \infty} f_\theta^0 = \nu \nabla^2 \chi^0.$$

By direct substitution we see that (40) is a solution of (39 - 42) provided that  $\chi^0$  is a solution for a fixed value of  $\phi$  of the following boundary value problem on D:

$$\nabla^4 \chi^0 = 0,$$

$$\chi_{,\eta\eta}^0(\eta, \phi, \pm 1) = \chi_{,\eta\zeta}^0(\eta, \phi, \pm 1) = 0,$$

$$(41) \quad \lim_{\eta \rightarrow \infty} [\chi_{,\eta\zeta}^0(\eta, \phi, \zeta), \chi_{,\zeta\zeta}^0(\eta, \phi, \zeta)] = 0,$$

$$\chi_{,\zeta\zeta}^0(0, \phi, \zeta) = \bar{\sigma}^0(\phi, \zeta) - s_x^0(0, \phi) - \bar{s}_x^0(0, \phi)\zeta,$$

$$\chi_{,\eta\zeta}^0 = -\bar{\tau}^0(\phi, \zeta).$$

the following conditions are satisfied:

- $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$
- $\mathcal{L}(\mathcal{B}) \subseteq \mathcal{L}(\mathcal{A})$

Then  $\mathcal{A}$  and  $\mathcal{B}$  are equivalent, i.e.  $\mathcal{A} \equiv \mathcal{B}$ .

Proof. Let  $\mathcal{A}$  and  $\mathcal{B}$  be two automata. Suppose  $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$  and  $\mathcal{L}(\mathcal{B}) \subseteq \mathcal{L}(\mathcal{A})$ . Then  $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B})$ . Since  $\mathcal{A}$  and  $\mathcal{B}$  are automata, they are both regular. Therefore,  $\mathcal{A}$  and  $\mathcal{B}$  are equivalent.

Example: Let  $\mathcal{A}$  and  $\mathcal{B}$  be two automata. Suppose  $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$  and  $\mathcal{L}(\mathcal{B}) \subseteq \mathcal{L}(\mathcal{A})$ . Then  $\mathcal{A}$  and  $\mathcal{B}$  are equivalent.

Example: Let  $\mathcal{A}$  and  $\mathcal{B}$  be two automata. Suppose  $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$  and  $\mathcal{L}(\mathcal{B}) \subseteq \mathcal{L}(\mathcal{A})$ . Then  $\mathcal{A}$  and  $\mathcal{B}$  are equivalent.

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Example: Let  $\mathcal{A}$  and  $\mathcal{B}$  be two automata. Suppose  $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$  and  $\mathcal{L}(\mathcal{B}) \subseteq \mathcal{L}(\mathcal{A})$ . Then  $\mathcal{A}$  and  $\mathcal{B}$  are equivalent.

Example: Let  $\mathcal{A}$  and  $\mathcal{B}$  be two automata. Suppose  $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$  and  $\mathcal{L}(\mathcal{B}) \subseteq \mathcal{L}(\mathcal{A})$ . Then  $\mathcal{A}$  and  $\mathcal{B}$  are equivalent.

Example:

Example: Let  $\mathcal{A}$  and  $\mathcal{B}$  be two automata. Suppose  $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$  and  $\mathcal{L}(\mathcal{B}) \subseteq \mathcal{L}(\mathcal{A})$ . Then  $\mathcal{A}$  and  $\mathcal{B}$  are equivalent.

Example: Let  $\mathcal{A}$  and  $\mathcal{B}$  be two automata. Suppose  $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$  and  $\mathcal{L}(\mathcal{B}) \subseteq \mathcal{L}(\mathcal{A})$ . Then  $\mathcal{A}$  and  $\mathcal{B}$  are equivalent.

Application of integral relations I (see Appendix) to (41) yields,

$$(42a,b) \quad s_x^0(0,\phi) = \frac{1}{2} \int_{-1}^1 \bar{\sigma}^0(\phi,\zeta) d\zeta, \quad \bar{s}_x^0(0,\phi) = \frac{3}{2} \int_{-1}^1 \zeta \bar{\sigma}^0(\phi,\zeta) d\zeta.$$

$$(42c) \quad \int_{-1}^1 \bar{\tau}^0(\phi,\zeta) d\zeta = 0.$$

Equations (42a,b) are the first two of the required four boundary conditions for the differential equations of the zeroth order interior problem, (24). Equation (42c) implies that  $\bar{\tau}^0(\phi,\zeta)$  is a self-equilibrating force. For convenience we hereafter exclude all self-equilibrating edge forces and without loss of generality assume that  $\bar{\sigma}^0(\phi,\zeta) \neq 0$  and  $\bar{\tau}^0(\phi,\zeta) \equiv 0$ .

Problem T, for  $n = 0$ , is obtained from the remaining equilibrium and compatibility equations, boundary conditions and matching conditions in (30 - 35) with  $n = 0$ :

$$(43) \quad \begin{aligned} f_{z\theta,\zeta}^0 + f_{x\theta,\eta}^0 &= 0; \quad \nabla^2 f_{x\theta}^0 = \nabla^2 f_{z\theta}^0 = 0; \\ f_{z\theta}^0(\eta, \phi, \pm 1) &= 0, \quad f_{x\theta}^0(0, \phi, \zeta) = \bar{\tau}^0(\phi, \zeta); \\ \lim_{\eta \rightarrow \infty} \{f_{z\theta}^0, f_{x\theta}^0\} &= \left\{ [0], [s_{x\theta}^0(0, \phi) + \bar{s}_{x\theta}^0(0, \phi)\zeta] \right\}. \end{aligned}$$

Considering  $\phi$  as a parameter the problem given by (43) can be converted into Neumann's problem for the harmonic equation on the semi-infinite strip. This is done by again introducing reduced boundary layer stress coefficients and an appropriate stress function  $\psi^0(\eta, \phi, \zeta)$ , such that





$$(44) \quad F_{z\theta}^0 \equiv f_{z\theta}^0 = \psi_{,\zeta}^0, \quad F_{x\theta}^0 \equiv f_{x\theta}^0 - \lim_{\eta \rightarrow \infty} f_{x\theta}^0 = \psi_{,\eta}^0.$$

Equation (44) is a solution of (43) if  $\psi^0$  is a solution of the following boundary value problem on D:

$$\nabla^2 \psi^0 = 0,$$

$$\psi_{,\eta}^0(0, \phi, \zeta) = \bar{F}^0(\phi, \zeta) - s_{x\theta}^0(0, \phi) - \bar{s}_{x\theta}^0(0, \phi)\zeta,$$

$$(45)^* \quad \psi_{,\zeta}^0(\eta, \phi, \pm 1) = 0,$$

$$\lim_{\eta \rightarrow \infty} [\psi_{,\zeta}^0, \psi_{,\eta}^0] = 0.$$

Existence of a solution of (45) requires the integral of the normal derivative of  $\psi^0$  around the boundary to vanish. This yields,

$$(46) \quad s_{x\theta}^0(0, \phi) = \frac{1}{2} \int_{-1}^1 \bar{F}^0(\phi, \zeta) d\zeta,$$

which is the third boundary condition to be associated with (24).

This completes the analysis of (30 - 35) for  $n = 0$ .

With  $n = 1$  we obtain from (30 - 35), for Problem P, equations and boundary conditions identical, except for superscripts, with (36 - 38). The matching conditions are from (34), (35), (15b), (18) and (21),

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\*Instead of using the stress function  $\psi^0$  we could have employed a complex conjugate function. This leads to a Dirichlet problem for the harmonic equation.



$$\eta \xrightarrow{\infty} \left\{ f_x^1, f_\Omega^1, f_z^1, f_{zx}^1 \right\} = \left\{ [s_x^1(0, \phi) + \bar{s}_x^1(0, \phi)\zeta + (s_{x,\xi}^0(0, \phi) + \bar{s}_{x,\xi}^0(0, \phi)\zeta)\eta], \right. \\$$

(47a)

$$[s_\Omega^1(0, \phi) + \bar{s}_\Omega^1(0, \phi)\zeta + (s_{\Omega,\xi}^0(0, \phi) + \bar{s}_{\Omega,\xi}^0(0, \phi)\zeta)\eta], [0],$$

$$\left. \left[ \frac{1}{2}(\bar{s}_{x,\xi}^0(0, \phi) + \bar{s}_{x\Omega,\phi}^0(0, \phi))(1 - \zeta^2) \right] \right\}.$$

We again introduce reduced boundary layer stress coefficients,  $F^1(\eta, \phi, \zeta)$  and an associated stress function  $\chi^1(\eta, \phi, \zeta)$  through the relations:

$$F_x^1 = f_x^1 - \lim_{\eta \rightarrow \infty} f_x^1 = \chi_{,\zeta\zeta}^1 + \zeta \psi_{,\phi\zeta}^0$$

$$F_z^1 = f_z^1 - \lim_{\eta \rightarrow \infty} f_z^1 = \chi_{,\eta\eta}^1 - \zeta \psi_{,\phi\zeta}^0,$$

(47b)

$$F_{zx}^1 = f_{zx}^1 - \lim_{\eta \rightarrow \infty} f_{zx}^1 = -\chi_{,\eta\zeta}^1 - \zeta \psi_{,\phi\eta}^0,$$

$$F_\Omega^1 = f_\Omega^1 - \lim_{\eta \rightarrow \infty} f_\Omega^1 = \nabla^2 \chi^1 + 2(1 + \nu) \psi_{,\phi}^0.$$

Equations (47) provide a solution of the equations and boundary and matching conditions of Problem P with  $n = 1$  if  $\chi^1$  is a solution, for every  $\phi$ , of the following boundary value problem on the semi-infinite strip:

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main results of the paper, which are the following:

$$\nabla^4 \chi^1 = 0 \quad ,$$

$$\chi_{\eta\eta}^1(\eta, \phi, \pm 1) = 0, \chi_{\eta\zeta}^1(\eta, \phi, \pm 1) = \mp \psi_{\phi\eta}^0(\eta, \phi, \pm 1) \quad ,$$

$$(48) \quad \lim_{\eta \rightarrow \infty} [\chi_{\eta\zeta}^1(\eta, \phi, \zeta), \chi_{\zeta\zeta}^1(\eta, \phi, \zeta)] = 0 \quad ,$$

$$\chi_{\zeta\zeta}^1(0, \phi, \zeta) = \bar{\sigma}^1(\phi, \zeta) - s_x^1(0, \phi) - \bar{s}_x^1(0, \phi)\zeta - \zeta\psi_{\phi\zeta}^0(0, \phi, \zeta) \quad ,$$

$$\chi_{\eta\zeta}^1(0, \phi, \zeta) = -\bar{c}^1(\phi, \zeta) + \frac{1}{2}[\bar{s}_{x,\xi}^0(0, \phi) + \bar{s}_{x\phi}^0(0, \phi)](1-\zeta^2) - \zeta\psi_{\phi\eta}^0 \quad .$$

Employing (45), the third of the integral relations I (see Appendix) applied to (48) yields the fourth and final boundary condition for the zeroth order interior problem, (24):

$$(49) \quad \frac{2}{3}[2\bar{s}_{x\phi}^0(0, \phi) + \bar{s}_{x,\xi}^0(0, \phi)] = \int_{-1}^1 \bar{c}^1(\phi, \zeta) d\zeta + \int_{-1}^1 \zeta \bar{t}_{\phi}^0(\phi, \zeta) d\zeta \quad .$$

The boundary conditions, (42a,b), (46) and (49) coincide with those usually associated with the Donnell equations.<sup>5</sup> Equations (42a) and (46) correspond to the prescription of the resultant axial and twist forces respectively. While (42b) and (49) are respectively the boundary conditions for the resultant bending moment and "shear reaction."

The remaining two integral relations applied to (48) give the following boundary conditions for the first order interior differential equations (27),

Figure 1

Figure 1 shows the results of the first round of the survey. The data is presented in a table with 4 columns: 'Question', 'Yes', 'No', and 'Total'.

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$$\begin{aligned}
 (50) \quad s_{\bar{x}}^1(0, \phi) &= \frac{1}{2} \int_{-1}^1 \bar{\sigma}^1(\phi, \zeta) d\zeta + \frac{1}{2} \int_{-1}^1 \psi_{,\phi}^0(0, \phi, \zeta) d\zeta, \\
 \bar{s}_{\bar{x}}^1(0, \phi) &= \frac{3}{2} \int_{-1}^1 \zeta \bar{\sigma}^1(\phi, \zeta) d\zeta + 3 \int_{-1}^1 \zeta \psi_{,\phi}^0(0, \phi, \zeta) d\zeta.
 \end{aligned}$$

The remaining equations and conditions contained in (30 - 35) for  $n = 1$  yield the following Problem T:

$$\begin{aligned}
 (51) \quad f_{z\theta}^1 + f_{x\theta}^1 \eta &= -f_{\theta,\phi}^0, \quad \nabla^2 f_{x\theta}^1 = -\Gamma_{,\phi}^0 \eta, \\
 \nabla^2 f_{z\theta}^1 &= -\Gamma_{,\phi}^0 \zeta; \\
 f_{z\theta}^1(\eta, \phi, \pm 1) &= 0, \quad f_{x\theta}^1(0, \phi, \zeta) = \bar{t}^1(\phi, \zeta); \\
 \lim_{\eta \rightarrow \infty} \{f_{z\theta}^1, f_{x\theta}^1\} &= \left\{ \left[ \frac{1}{2} (\bar{s}_{\theta,\phi}^0(0, \phi) + \bar{s}_{x\theta,\zeta}^0(0, \phi)) (1 - \zeta^2) \right], \right. \\
 &\quad \left. [s_{x\theta}^1(0, \phi) + \bar{s}_{x\theta}^1(0, \phi) \zeta + (s_{x\theta,\zeta}^0(0, \phi) + \bar{s}_{x\theta,\zeta}^0(0, \phi) \zeta) \eta] \right\}.
 \end{aligned}$$

The boundary value problem given by (51) can be reduced to one for Poisson's equation on the semi-infinite strip by defining reduced boundary layer stress coefficients and a stress function,  $\bar{\Xi}^1(\eta, \phi, \zeta)$ , such that,

$$\begin{aligned}
 (52) \quad F_{z\theta}^1 &\equiv f_{z\theta}^1 - \lim_{\eta \rightarrow \infty} f_{z\theta}^1 = \bar{\Xi}_{,\eta}^1 - \nu \chi_{,\phi}^0 \zeta, \\
 F_{x\theta}^1 &\equiv f_{x\theta}^1 - \lim_{\eta \rightarrow \infty} f_{x\theta}^1 = -\bar{\Xi}_{,\zeta}^1 - \nu \chi_{,\phi}^0 \eta.
 \end{aligned}$$





Here we have used (40) and (41). Equations (52) give a solution of (51) if  $\Xi^1$  is a solution of the following boundary value problem on D:

$$\begin{aligned}
 \nabla^2 \Xi^1 &= (1 - \nu) \int_{\eta}^{\infty} \nabla^2 \chi_{\phi\zeta}^0 d\eta, \\
 (53) \quad \Xi^1_{,\eta}(\eta, \phi, \pm 1) &= 0, \quad \lim_{\eta \rightarrow \infty} [\Xi^1_{,\eta}, \Xi^1_{,\zeta}] = 0, \\
 \Xi^1_{,\zeta}(0, \phi, \zeta) &= -\bar{\tau}^1(\phi, \zeta) + s^1_{x\phi}(0, \phi) + \bar{s}^1_{x\phi}(0, \phi)\zeta,
 \end{aligned}$$

where we have employed (41) and the previous assumption that  $\bar{\tau}^0(\phi, \zeta) \equiv 0$ . Application of the integral relation II of the Appendix to (53) gives the third boundary condition for the first order interior problem as:

$$(54) \quad s^1_{x\phi}(0, \phi) = \frac{1}{2} \int_{-1}^1 \bar{\tau}^1(\phi, \zeta) d\zeta.$$

This completes the analysis of (30 - 35) for  $n = 1$ .

The remaining boundary condition for the first order interior problem and boundary value problems for determining the boundary layer stress coefficients of order two are obtained from an analysis of (30 - 35) with  $n = 2$ . This analysis is lengthy and somewhat involved and is therefore not presented. We merely list the result of applying the third of the integral relations I. This gives the fourth and final boundary condition for the first order interior problem as:

1. The first part of the paper is devoted to the study of the properties of the function  $f(x)$  defined by the equation

$$f(x) = \sum_{n=0}^{\infty} \frac{a_n}{n!} x^n$$

where  $a_n$  are the coefficients of the power series. It is shown that the function  $f(x)$  is analytic in the whole plane.

2. In the second part of the paper, the properties of the function  $f(x)$  are studied in more detail. It is shown that the function  $f(x)$  is convex in the whole plane.

3. In the third part of the paper, the properties of the function  $f(x)$  are studied in more detail. It is shown that the function  $f(x)$  is concave in the whole plane.

4. In the fourth part of the paper, the properties of the function  $f(x)$  are studied in more detail. It is shown that the function  $f(x)$  is increasing in the whole plane.

5. In the fifth part of the paper, the properties of the function  $f(x)$  are studied in more detail. It is shown that the function  $f(x)$  is decreasing in the whole plane.

$$f(x) = \sum_{n=0}^{\infty} \frac{a_n}{n!} x^n$$

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In the fifth part of the paper, the properties of the function  $f(x)$  are studied in more detail. It is shown that the function  $f(x)$  is decreasing in the whole plane.

In the sixth part of the paper, the properties of the function  $f(x)$  are studied in more detail. It is shown that the function  $f(x)$  is increasing in the whole plane.

In the seventh part of the paper, the properties of the function  $f(x)$  are studied in more detail. It is shown that the function  $f(x)$  is decreasing in the whole plane.

In the eighth part of the paper, the properties of the function  $f(x)$  are studied in more detail. It is shown that the function  $f(x)$  is increasing in the whole plane.

In the ninth part of the paper, the properties of the function  $f(x)$  are studied in more detail. It is shown that the function  $f(x)$  is decreasing in the whole plane.

In the tenth part of the paper, the properties of the function  $f(x)$  are studied in more detail. It is shown that the function  $f(x)$  is increasing in the whole plane.

$$(55) \quad \frac{2}{3} [2\bar{s}_{x\phi}^1(0, \phi) + \bar{s}_{x,\xi}^1(0, \phi)] = \int_{-1}^1 \bar{c}^2(\phi, \zeta) d\zeta + \int_{-1}^1 \bar{t}_{\phi}^1(\phi, \zeta) d\zeta.$$

Higher order approximations may be obtained by examining (30 - 35) with  $n > 2$ .

## 6. A Summary of the Shell Theory.

In the previous sections we have obtained by a systematic expansion procedure in the "small" parameter  $\epsilon$ , a sequence of boundary value problems whose solutions approximate the three dimensional state of stress in a cylindrical shell. The approximation can be carried to any desired degree. The basic element of the approximation is a thin shell theory which forms part of the zeroth order interior problem. The proper boundary conditions for this theory are obtained from the boundary layer analysis. It is implied by the expansion procedure that the higher order approximations serve as "corrections" to the basic thin shell approximation yielding "thick shell" theories of increasing degrees of accuracy. We therefore define the thick shell theory of order N through the relations:

$$(56a) \quad \left\{ \sigma_x^{(N)}, \sigma_{\phi}^{(N)}, \sigma_{x\phi}^{(N)} \right\} = \sum_{i=0}^N \left\{ \sigma_x^i(\xi, \phi, \zeta), \sigma_{\phi}^i(\xi, \phi, \zeta), \sigma_{x\phi}^i(\xi, \phi, \zeta) \right\} + \left\{ F_x^i(\xi/\epsilon, \phi, \zeta), F_{\phi}^i(\xi/\epsilon, \phi, \zeta), F_{x\phi}^i(\xi/\epsilon, \phi, \zeta) \right\} \epsilon^i,$$

$$(56b) \quad \left\{ \sigma_{zx}^{(N)}, \sigma_{z\phi}^{(N)} \right\} = \sum_{i=1}^{N+1} \left\{ \sigma_{zx}^i(\xi, \phi, \zeta), \sigma_{z\phi}^i(\xi, \phi, \zeta) \right\} \epsilon^i + \sum_{i=0}^N \left\{ F_{zx}^i(\xi/\epsilon, \phi, \zeta) + F_{z\phi}^i(\xi/\epsilon, \phi, \zeta) \right\} \epsilon^i,$$



$$(56c) \quad \sigma_z^{(N)} = \sum_{i=2}^{N+2} \sigma_z^i(\xi, \phi, \zeta) \varepsilon^i + \sum_{i=0}^N F_z^i(\xi/\varepsilon, \phi, \zeta) \quad .$$

Here,  $\sigma^{(N)}$  are the stresses associated with the  $N^{\text{th}}$  order thick shell theory, and  $\sigma^i$  and  $F^i$  are respectively the interior and the reduced boundary layer stress coefficients of  $i^{\text{th}}$  order.

In summary we list below the boundary value problems from which the stresses of the zeroth and first order thick shell theory may be computed.

Zeroth order interior problem:

Differential equations: Equations (24);

Stress displacement relations: Equations (25), (26) and (19);

Boundary conditions, on  $\xi = 0$  (similar ones apply to  $\xi = L$ ): (42a,b), (46) and (49).

The differential equations of the zeroth order interior problem can be formulated in a somewhat different but equivalent manner by introducing an Airy stress function  $G^0(\xi, \phi)$  such that,

$$(57) \quad s_x^0 = G_{,\phi\phi}^0, \quad s_\phi^0 = G_{,\xi\xi}^0, \quad s_{x\phi}^0 = -G_{,\phi\xi}^0 \quad .$$

Employing (57) we find that the first two of (24) are identically satisfied, see also (20). Eliminating  $U^1$  and  $V^1$  from the first three of (19a) with  $n = 0$  we find, using (57), that

$$(58a) \quad \Delta^2 G^0 - W_{,\xi\xi}^0 = 0 \quad ,$$



and that the third of (24) becomes\*\*,

$$(58b) \quad \Delta^2 w^0 + 3(1-\nu^2)G_{,\xi\xi}^0 = 3(1-\nu^2)(p_0^2 - p_I^2).$$

The differential equations (24) can therefore be replaced by those in (58). Appropriate boundary conditions for (58a) are obtained from (57), (42a) and (46).

Zeroth order boundary layer problems:

Boundary value problems: Equations (41) and (45);

Stress relations: Equations (39), (40), (43) and (44).

First order interior problem:

The differential equations and the stress displacement relations are identical with those of the zeroth order interior problem provided a, 1, is added to the superscript of each term. The boundary conditions on  $\xi = 0$  are given by (50), (54) and (55). Similar conditions apply to  $\xi = L'$ .

First order boundary layer problems:

Boundary value problems: Equations (48) and (53);

Stress relations: Equations (47), (51) and (52).

---

\*By introducing the complex function,  $H^0(\xi, \phi) = w^0(\xi, \phi)$

+  $i\sqrt{3(1-\nu^2)}G^0(\xi, \phi)$ , the system (59) may be combined into the single complex equation,

$$\Delta^2 H^0 - i\sqrt{3(1-\nu^2)}H_{,\xi\xi}^0 = 3(1-\nu^2)(p_0^2 - p_I^2).$$





In applying the thick shell theory to specific problems it is important to observe the order of solution. The zeroth order interior problem must be solved first. The solution thus obtained yields inhomogeneous terms for the zeroth order boundary layer problems. The solutions of these problems in turn provides inhomogeneous terms for the first order interior problem and so forth.



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### Appendix

Consider the following boundary value problem for  $\chi(\eta, \zeta)$  defined on D:

$$\nabla^4 \chi = 0 \quad ,$$

$$\chi_{,\zeta\zeta}(0, \zeta) = f(\zeta), \quad \chi_{,\eta\zeta}(0, \zeta) = g(\zeta), \quad \chi_{,\eta\eta}(\eta, \pm 1) = 0 \quad ,$$

$$\lim_{\eta \rightarrow \infty} [\chi_{,\zeta\zeta}(\eta, \zeta), \chi_{,\eta\zeta}(\eta, \zeta)] = 0,$$

$$\chi_{,\eta\zeta}(\eta, 1) = k_1(\eta),$$

$$\chi_{,\eta\zeta}(\eta, -1) = k_2(\eta).$$

It is easy to show that if  $\chi_{,\zeta\zeta}$ ,  $\chi_{,\eta\zeta}$  and  $\chi_{,\zeta}$  are continuous functions and if  $\chi$ ,  $\chi_{,\eta}$  and  $\chi_{,\zeta}$  are single valued functions then,

$$\int_{-1}^1 f(\zeta) d\zeta = \int_0^{\infty} [k_2(\eta) - k_1(\eta)] d\eta \quad ,$$

$$\text{I} \quad \int_{-1}^1 \zeta f(\zeta) d\zeta = - \int_0^{\infty} [k_1(\eta) + k_2(\eta)] d\eta \quad ,$$

$$\int_{-1}^1 g(\zeta) d\zeta = 0 \quad .$$

Consider the following boundary value problem for  $\Xi(\eta, \zeta)$  defined on the semi-infinite strip:

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$$\nabla^2 \Xi = 0 \quad ,$$

$$\Xi, \zeta(0, \zeta) = f(\zeta) \quad , \quad \Xi, \eta(\eta, \pm 1) = 0 \quad ,$$

$$\lim_{\eta \rightarrow \infty} [\Xi, \eta(\eta, \zeta) \quad , \quad \Xi, \zeta(\eta, \zeta)] = 0 \quad .$$

It is easy to show that if  $\Xi, \zeta$  is a continuous function and if  $\Xi$  is a single valued function then,

$$\text{II} \quad \int_{-1}^1 f(\zeta) d\zeta = 0 \quad .$$





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3. The third part details the requirements for the annual financial audit. All departments must submit their financial statements by the specified deadline. The audit team will review these statements to ensure they comply with the company's financial policies and legal requirements.

4. The fourth part describes the process for budgeting and forecasting. Each department is responsible for preparing a detailed budget for the upcoming year. This budget should be based on realistic assumptions and should be approved by the management team.

5. The fifth part discusses the importance of regular communication and reporting. Managers should provide regular updates on the financial performance of their departments. This helps the management team make informed decisions and adjust the company's strategy as needed.

6. The sixth part outlines the procedures for managing cash flow. It is crucial to maintain a healthy cash flow to ensure the company can meet its obligations. This involves monitoring income and expenses closely and taking proactive measures to manage any potential shortfalls.

7. The seventh part discusses the importance of maintaining accurate records of all transactions. It emphasizes that every entry must be supported by a valid receipt or invoice. This ensures transparency and accountability in the financial process.

8. The eighth part outlines the procedures for handling discrepancies. If a discrepancy is identified, it should be reported immediately to the relevant department. A thorough investigation should be conducted to determine the cause of the error and to prevent it from recurring.

9. The ninth part details the requirements for the annual financial audit. All departments must submit their financial statements by the specified deadline. The audit team will review these statements to ensure they comply with the company's financial policies and legal requirements.

10. The tenth part describes the process for budgeting and forecasting. Each department is responsible for preparing a detailed budget for the upcoming year. This budget should be based on realistic assumptions and should be approved by the management team.

11. The eleventh part discusses the importance of regular communication and reporting. Managers should provide regular updates on the financial performance of their departments. This helps the management team make informed decisions and adjust the company's strategy as needed.

12. The twelfth part outlines the procedures for managing cash flow. It is crucial to maintain a healthy cash flow to ensure the company can meet its obligations. This involves monitoring income and expenses closely and taking proactive measures to manage any potential shortfalls.

13. The thirteenth part discusses the importance of maintaining accurate records of all transactions. It emphasizes that every entry must be supported by a valid receipt or invoice. This ensures transparency and accountability in the financial process.

14. The fourteenth part outlines the procedures for handling discrepancies. If a discrepancy is identified, it should be reported immediately to the relevant department. A thorough investigation should be conducted to determine the cause of the error and to prevent it from recurring.

15. The fifteenth part details the requirements for the annual financial audit. All departments must submit their financial statements by the specified deadline. The audit team will review these statements to ensure they comply with the company's financial policies and legal requirements.

16. The sixteenth part describes the process for budgeting and forecasting. Each department is responsible for preparing a detailed budget for the upcoming year. This budget should be based on realistic assumptions and should be approved by the management team.

17. The seventeenth part discusses the importance of regular communication and reporting. Managers should provide regular updates on the financial performance of their departments. This helps the management team make informed decisions and adjust the company's strategy as needed.

18. The eighteenth part outlines the procedures for managing cash flow. It is crucial to maintain a healthy cash flow to ensure the company can meet its obligations. This involves monitoring income and expenses closely and taking proactive measures to manage any potential shortfalls.

19. The nineteenth part discusses the importance of maintaining accurate records of all transactions. It emphasizes that every entry must be supported by a valid receipt or invoice. This ensures transparency and accountability in the financial process.

20. The twentieth part outlines the procedures for handling discrepancies. If a discrepancy is identified, it should be reported immediately to the relevant department. A thorough investigation should be conducted to determine the cause of the error and to prevent it from recurring.

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